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CONSTRUCTION AND OPTIMIZATION OF A MULTIFACTOR EXPERIMENT IN THE PRESENCE OF CONSTRAINTS

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Special features of construction of an experimental multifactor model in the presence of constraints have been considered. The conditions of formation of constraints in the factor space and the search for the Y(X) extremum on the subset X of admissible combinations of factor levels belonging to the region Ω have been distinguished.

An experimental mathematical model is a symbolic representation that expresses, with a required accuracy, the quantitative relations characterizing the object under investigation. The present work suggests a method of construction of an analytical model where the relation between the levels of the factors and the response can be expressed by one of the combinations of functions of the assumed forms (rational, linear fractional, exponential, logarithmic, etc.)

The method of steepest descent, rise over a crest, and simplex-planning of an experiment are used to find the extremum in multifactor problems. The search for the optimum in multifactor problems includes the following stages: 1) problem formulation; 2) collection of *a priori* information; 3) pre-planning an experiment (those factors are distinguished that can affect object functioning and the optimization parameter); 4) planning and implementation of the experiment at the initial point of search; 5) search for the optimum region; 6) planning an experiment in the optimum region. The present study is aimed at determining a plan for constructing a model in the local region that encompasses the optimum point [1, 2].

The problem suggested is solved by a modified method: construction and optimization of the experimental mathematical model are executed on the basis of statistical data in the presence of functional constraints

$$a_{i\min} \le \varphi_i(x_1, x_2, ..., x_k) \le a_{i\max} ,$$

with the form of the constraint function and its parameters being determined based on experimental data. In the general case, the constraints determine the region Ω of admissible combinations of the levels of factors in the factor space. Consequently, the problem of optimization in the presence of constraints can be formulated mathematically as the problem of the search for the $Y(X)$ extremum on the subset of values of X belonging to Ω.

At the first stage, mathematical models for the parameters of optimization and constraints are constructed; then a conventional extremum, the models of whose constraints are the equations of coupling between the levels of the factors, is determined analytically. Thus, the problem is reduced to determination of the conventional extremum by the Lagrange multiplier method, the essence of which is the following: the response function is determined as the polynomial model

$$y = f(x_1, x_2, ..., x_k) \tag{1}$$

provided that the k factors are related by n ($n < k$) equations that are parametrically nonlinear:

$$Z_1 = \varphi_1(x_1, x_2, ..., x_k) ,$$

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TABLE 1. Planning Matrix and Experimental Results

Variation range and level of the factors	x_0	x_1 (δ)	x_2 (v, km/h)	x_1x_2	z
Main level (0)		0.5	32		
Variation ranges		0.1	16		
Lower level (–1)		0.4	16		
Upper level (+1)		0.6	48		
1	+	+	+	+	0.78
2	+	—	+	—	0.79
3	+	+	—	—	0.81
4	+	—	—	+	0.82
5	+	+	+	+	0.81
6	+	—	+	—	0.81
7	+	+	—	—	0.80
8	+	—	—	+	0.79

$$Z_2 = \varphi_2(x_1, x_2, ..., x_k) ,$$

$$.......................... ,\tag{2}$$

$$Z_n = \varphi_n(x_1, x_2, ..., x_k) .$$

We compose the Lagrange function

$$F(x_1, x_2, ..., x_k, \lambda_j) = f(x_1,x_2, ..., x_k) + \lambda_1(\varphi_1(x_1,x_2, ..., x_k) — z_1) +$$

$$+ \lambda_2(\varphi_2(x_1,x_2, ..., x_k) — z_2) + ... + \lambda_n(\varphi_n(x_1,x_2, ..., x_k) — z_k) .\tag{3}$$

Having equated its partial derivatives with respect to all factors and multipliers λ, we obtain a system of $n + k$ equations of the form

$$\frac{\partial F}{\partial x_i} = \frac{\partial f}{\partial x_i} + \sum_{j=1}^{n} \lambda_j \frac{\partial \varphi_n}{\partial x_i} = 0 ,\tag{4}$$

$$\frac{\partial F}{\partial \lambda_j} = \varphi_n(x_1,x_2, ..., x_k) — z_k = 0 ,\tag{5}$$

from which we find the values of x_i and λ_j.

We determine the necessary conditions for extremum existence by Eqs. (4) and (5) and the sufficient conditions of its existence by the sign of the total differential of second order: $d^2F > 0$ at the point where X is minimum and $d^2F < 0$ where it is maximum. We should mention special features of construction of the Lagrange function. Function (1) is constructed based on the fractional factor experiment 2^k. Coupling equations (2) are written proceeding from the shape of the curve of experimental data in construction of which rectification (leveling) by corresponding substitution of coordinates is done. The method of leveling depends on the type of the model and the number of its parameters. A preliminary decision on the applicability of the selected model is made if in the system of coordinates such a curve can be drawn to which experimental points are very close.

This technique was used for constructing a nonlinear statistical model of an engineering problem that has the form of (3)

$$F(x_1, x_2, \lambda) = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_1 x_2 + \lambda \left(c_0 + c_1 x_2 + c_2 x_2^2 - \frac{x_1}{b_0 + b_1 x_1^2} \right),$$

of variation of the braking torque (constraint) and the value of the coefficient of longitudinal adhesion of wheels (δ) as a function of the velocity of motion (v).

To obtain mathematical models in the form of a polynomial, we realized the fractional factor experiment 2^3. As the factors we consider the velocity of vehicle motion v (km/h) and the value of the coefficient of longitudinal adhesion of the wheels δ. Table 1 gives the levels of the factors and experimental results.

The following model was obtained based on processing of the results:

$$F(x_1, x_2, \lambda) = 0.8 - 1.25 \cdot 10^{-3} x_1 - 3.75 \cdot 10^{-3} x_2 - 0.2 x_1 x_2 +$$

$$+ \lambda \left(-6.4 \cdot 10^{-4} + 3.2 \cdot 10^{-3} x_2 - 5.1 \cdot 10^{-5} x_2^2 - \frac{x_1}{0.2919 + 1.149 x_1^2} \right).$$

Differentiation of F with respect to x_1, x_2, and λ, provided that the values of x_1 and x_2 are on the surface of the radius ρ ($\rho = x_1^2 + x_2^2$), yields a system of three nonlinear equations. Results of solution of this system give the following optimum values of the levels of the factors: δ = 0.63 and v = 56 km/h. Moreover, the data obtained are the constraints in construction of an optimal multimass vibrational system from the point of view of the minimum of root-mean-square values of mass acceleration.

Thus, based on the technique developed, we determined optimum values of one of the response functions with constraints. This made it possible to analytically estimate the exploitation properties (factors v and δ) of a vehicle.

NOTATION

a_0, a_1, a_2, a_3, c_0, c_1, and c_2, coefficients of the polynomial model; b_0 and b_1, coefficients of the coupling equation; v, car velocity; Z_n, constraint functions; $Z_k \in Z_n$, values of the factor belonging to the constraint region; λ_n, Lagrange multiplier; δ, coefficient of longitudinal adhesion of the wheels; ρ, response surface. Indices: max, maximum; min, minimum.

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